Real-Time Sub-cm Differential Orbit Determination of Two Low-Earth Orbiters with GPS Bias Fixing

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BIOGRAPHY

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ABSTRACT

An effective technique for real-time differential orbit determination of two low Earth orbiters with GPS bias fixing is formulated. With this technique, only moderate-quality GPS orbit and clock states (e.g., as available in real-time from the NASA Global Differential GPS System with 10–20 cm accuracy) are needed to seed the process. The onboard, real-time orbital states of user satellites (few meters in accuracy) are used for orbit initialization and integration. An extended Kalman filter is constructed for the estimation of the differential orbit between the two satellites as well as a reference orbit, together with their associating dynamics parameters. The technique assumes that the two satellites are separated by a moderately long baseline (hundreds of km or less), and that they are of roughly similar shape. The differential dynamics, therefore, can be tightly constrained, strengthening the orbit determination. Without explicit differencing of GPS data, double-differenced phase biases are formed by a special transformation matrix. Integer-valued fixing of these biases is then performed, greatly improving the orbit estimation. A 9-day demonstration with the two GRACE spacecraft (with baselines of ~200 km) indicates that ~80% of the double-differenced phase biases can be successfully fixed, and the differential orbit can be determined to ~7 mm 1D RMS as compared to direct measurements of the micron-precision, onboard K-band ranging sub-system.

INTRODUCTION

Constellations of Earth orbiting satellites are of increasing interest in earth science observation and military applications. Two or more relatively small satellites flying in formation provide a low-cost substitute for single large satellite, with enhanced flexibility and redundancy.

Among the key issues with formation flying is the knowledge of the relative position between the satellites. Although precise relative orbits are required only after the fact under most civil applications, some missions will benefit from real-time (or near real-time) precise orbits. Examples of such missions include bi-static radar, virtual antennae in space, and space-based radar constellations. These missions require real-time knowledge of baselines between two or more spacecraft either for precise formation control or for onboard science processing and data compression.

Since the inception of the U.S. Global Positioning System (GPS), applications to orbit determination of low earth orbiters (LEO's) have been investigated and demonstrated. GPS based orbit accuracy has been steadily improving, from several cm with Topex/Poseidon [1] to the 1-2 cm with GRACE [2] and sub-centimeter for Jason [3]. Such accuracies have been obtained with precision GPS receivers tracking dual-frequency (for the removal of ionosphere effects) GPS pseudorange and carrier phase measurements, using ground-based post-processing software.

GPS carrier phase is a precise measurement of the sum of the clock offset and the distance between the transmitter (GPS) and the receiver (user). However, the measurement only accounts for the continuous change in phase, lacking a knowledge of its absolute value. Each set of continuous phase measurements has a constant bias over the entire continuous pass for the transmitter-receiver pair. In applications using carrier phases, such unknown biases have to be estimated as real numbers together with other parameters, thus weakening the data strength and limiting the ultimate GPS positioning accuracy. Integer-valued determination of the number of wavelengths in the phase biases (also known as resolving phase ambiguities, or bias-fixing) has long been attempted and has demonstrated to greatly strengthen ground positioning accuracy with GPS. A review of many researchers' works in this respect has been reported in [4].

An application of GPS phase bias fixing to differential orbit determination has been demonstrated to millimeter accuracy [5] between the two earth orbiting satellites of GRACE, separated by ~200 kilometers. The technique requires predetermination of the GRACE orbits to ~2 cm accuracy, and the GPS orbits to ~5 cm accuracy. In real-time applications, such high-accuracy predetermined orbits are not available and bias fixing has been rendered difficult, if not impossible. The current accuracy of real-time differential orbit determination between LEO's is of the order of several centimeters [6,7].

This paper reports an effective technique for real-time differential orbit determination with GPS bias fixing. With this technique, only moderate-quality GPS orbits and clocks are needed (as available in real-time from the NASA Global Differential GPS System with 10–20 cm accuracy). For short baselines (< 100 km) a lower quality GPS orbit and clock states can be used, such as the broadcast ephemeris. The LEO onboard, real-time orbital states (up to few meters in accuracy) are used for orbit initialization and integration. Taking advantage of the close proximity of the two satellites, and of similar body shapes, differential dynamics between the two LEO's can be tightly constrained and the orbit estimation strengthened. An extended Kalman filter is constructed for the estimation of the *differential* orbit between the two satellites as well as a reference orbit, together with their associating dynamics parameters. Without explicit differencing of GPS data, double-differenced phase biases are formed by a transformation matrix. Integer-valued fixing of these biases are then performed which greatly strengthens the orbit estimation.

The technique is demonstrated with the GRACE orbits over a 9-day period in August, 2004. During this period the baseline between the two LEO's is ~200 km in length. The "length" component of the 3-dimensional baseline vector solution is compared with the precise K-band dual one-way ranging between the satellites, which is accurate to the microns level except for an unknown bias. The length agreement has an RMS value of ~7 mm over the 9-day period. It improves to ~4 mm when the baseline becomes shorter (35 to 65 km) during an orbit switchover operation in December 2005. In addition, the percentage of successfully fixed double-differenced phase biases can serve as a qualitative measure of the filtering process. A high success rate of ~80% is demonstrated.

THE LAMBDA METHOD FOR BIAS FIXING

Carrier phase biases are in general estimated (together with other parameters) as real (floating-point) valued parameters. Due to instrument delays in the transmitters and receivers, the biases are not integers in nature. Such unknown delays cancel out upon double differencing between two transmitters and two receivers, allowing integer-valued fixing of phase biases (or resolving of phase ambiguities).

One of the difficulties in resolving phase ambiguities is that the floating-point solutions of phase biases are mutually correlated in general. As a consequence of such correlation, the search space for correct integers is large, with low confidence in fixing. De-correlating these floating-point bias solutions would allow sequential conditional search for correct integers. This has become the key issue in bias fixing and the vast research in this area has led to various de-correlation techniques [4]. In this paper, a method pioneered by Teunissen [8], the LAMBDA (**L**east-squares **AMB**iguity **D**ecorrelation **A**djustment) method, is adopted for de-correlating the floating-point solutions of double-differenced phase biases and subsequently fixing them.

VALIDATION OF INTEGER-VALUE FIXED BIASES

An incorrectly fixed bias would result in erroneous orbit determination in a way more damaging to orbit determination than the un-fixed bias. Therefore, prior to incorporating into the orbit solution, each of the fixed biases needs to be validated. In the following, two validation tests will be carried out, the wide-lane bias test and the ionosphere-free bias test.

Let Δ_1 and Δ_2 be the differences between the fixed integer-valued solution of L1 and L2 biases, respectively, from their real-valued solutions. Then the wide-lane bias test is

$$\left|\Delta_1 - \Delta_2\right| < \alpha$$

and the ionosphere-free bias test is

$$\left|f_1^2\lambda_1\Delta_1 - f_2^2\lambda_2\Delta_2\right| / (f_1^2 - f_2^2) < \beta$$

where f_1 and f_2 are the frequencies of L1 and L2 signals, and λ_1 and λ_2 their corresponding wavelengths.

A trial and error approach has led to the choice $\alpha = 0.2$ cycle and $\beta = 1$ cm.

Aside from these two tests, bias fixing is not attempted when the formal error of the differential position estimate is higher than a threshold of 2 cm in any component.

DOUBLE-DIFFERECED PHASE BIASES

As mentioned above, only double-differenced phase biases are integer in nature and can be candidates for being fixed. Explicitly double differencing GPS measurements would of course result in double-differenced biases but is undesirable for two reasons. Firstly, The measurements now involves 2 transmitters and 2 receivers and the measurement partial derivatives become more complicated. Secondly, double-differenced GPS measurements are no longer uncorrelated and cannot be properly weighted without a complex weighting matrix.

We adopt here a UD-formulated sequential filtering process [9] on un-differenced GPS measurements. All measurements at a given epoch are processed as usual for the estimation of un-differenced phase biases and other parameters. The UD array (which contains the information of the estimates and their covariances) associated with the bias parameters is transformed into a UD array for double-differenced biases through a special mapping. It is a rectangular matrix of dimension N (rows) by M (columns), where M and N are the number of un-differenced and double-differenced biases, respectively. All but four elements in each row of the mapping matrix are 0, with the only non-zero elements of values 1, -1, -1 and 1. The matrix is formed one row at a time with an exhaustive search of non-redundant double-differenced combinations. This matrix effectively converts un-differenced biases to double-differenced biases. The GPS measurements remain undifferenced, uncorrelated and with simple partial derivatives with respect to estimated parameters.

UD UPDATE WITH FIXED BIASES

A unique property of a UD array (each row of which corresponds to an estimated parameter) is that any new information update of the parameters located at the lower part of the array will not affect the upper part of the array. With this property in mind, it is efficient to put the phase biases at the end of the estimated parameter list; its associated UD array will then reside at the bottom. With such arrangement, only the bottom part of the UD array will need to be transformed into that of double-differenced biases and to be updated with any subsequently fixed biases. The upper part of the UD array will remain unchanged by such transformation and updating.

The LAMBDA bias fixing algorithm is carried out on the transformed UD array to search for any fixable double-differenced biases. In general, only a subset of all the biases will get fixed to the correct integers with properly pre-specified confidence criteria. With each of the confidently fixed double-differenced biases, the original (un-transformed) UD array containing the un-differenced biases is updated by a pseudo-measurement defined by a single-row matrix equation. The left-hand side of this equation is a 1×M matrix with elements identical to the row of the transformation matrix forming that particular double-differenced bias. The right-hand side is the LAMBDA fixed integer value of the double-differenced bias. A large data weight of 10^6 cycle^{-1}, is assigned to reflect the error-free estimate of the fixed bias.

DIFFERENTIAL ORBIT AND DYNAMICS

Precise modeling of the dynamics controlling low earth orbiters is by no means a trivial task. Purely dynamic orbit determination of orbiters at ~400 km altitudes over a 24-hour period is limited to the decimeter level. For applications requiring higher orbit accuracy, a reduced-dynamic filtering scheme [10] is required. With this filtering scheme, empirical forces in all 3 directions are modeled as constrained process noise and estimated. The level of mismodeling of the overall dynamics dictates the level of the applied constraints. For a pair of neighboring satellites with similar body shape and flying along nearly the same path, the dynamics are highly common and the differential mismodeling of the dynamics is expected to be far lower than the mismodeling of the individual satellite's dynamics. Hence, the constraint on the process noise for differential dynamics can be greatly tightened and, consequently, the solution strengthened.

To exploit the high commonality of the differential dynamics we estimate the differential orbit instead of the individual orbits. In addition, a reference orbit (which is one of the individual orbits) needs to be estimated to avoid having both orbits drifts away over time.

Let $\mathbf{a}$ and $\mathbf{b}$ be the state vectors (3-d positions, velocities and dynamics parameters to be estimated) of the individual orbits A and B, respectively. Also, let $\mathbf{u}$ and $\mathbf{v}$

be the state vectors of the differential orbit and of the reference orbit, respectively. In other words,

$$\mathbf{u} = \mathbf{a} - \mathbf{b}$$

$$\mathbf{v} = \mathbf{b}$$

Using the chain rule, the measurement partial derivatives wrt. the orbital states, **u** and **v**, can be expressed in terms of those wrt. each individual orbital states, **a** and **b**, as,

$$\partial M / \partial \mathbf{u} = \partial M / \partial \mathbf{a}$$

$$\partial M / \partial \mathbf{v} = \partial M / \partial \mathbf{a} + \partial M / \partial \mathbf{b}$$

where M denotes a GPS measurement, either pseudorange or carrier phase.

The matrices of variational partial derivatives mapping the state vectors of the individual orbits between two time points, 1 and 2, are generated by integrations of the orbit dynamics. Let these matrices be $\partial A_2 / \partial A_1$ and $\partial B_2 / \partial B_1$, with the corresponding mapping equation

$$\begin{bmatrix} \mathbf{a}_2 \\ \mathbf{b}_2 \end{bmatrix} = \begin{bmatrix} \partial A_2 / \partial A_1 & 0 \\ 0 & \partial B_2 / \partial B_1 \end{bmatrix} \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{b}_1 \end{bmatrix}$$

Then, using the chain rule, the matrix of variational partial derivatives mapping the state vectors of the differential orbit and the reference orbit can be derived. The resulting mapping equation is

$$\begin{bmatrix} \mathbf{u}_2 \\ \mathbf{v}_2 \end{bmatrix} = \begin{bmatrix} \partial A_2 / \partial A_1 & \partial A_2 / \partial A_1 - \partial B_2 / \partial B_1 \\ 0 & \partial B_2 / \partial B_1 \end{bmatrix} \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{v}_1 \end{bmatrix}$$

Integrations of orbits are performed on each individual satellites. After processing GPS measurements and any possible bias fixing at each epoch, the part of the UD array associating with the state vectors **u** and **v** is mapped to the next epoch using the above mapping equation.

ESTIMATION PROCESS

This section outlines the overall estimation process for the differential orbit determination.

In addition to the state vectors **u** and **v** and GPS phase biases several other parameters must be estimated to assure precise orbit determination, as is discussed below.

The effects of ionosphere delays, if not properly removed, can be a limiting factor on precise orbit determination. In most GPS applications, measurements at two frequencies are combined to remove the ionosphere delay. However, such combination would destroy the integer nature of double-differenced phase biases and should be avoided. To facilitate bias fixing, un-combined GPS measurements at *both* L1 and L2 frequencies should be used. The effects of the ionosphere are removed by estimating line-of-sight delays at each epoch. These ionosphere delays are treated as white noise from one time to another and uncorrelated between observing links. The proper inversed-frequency-squared scaling between L1 and L2 measurements, and the negative delay on phase versus positive delays on pseudorange, are applied. Hence only single delay is to be estimated for each observing link.

The real-time onboard knowledge of differential clocks between the two user satellites is not sufficiently accurate for precise differential orbit determination and has to be estimated as a loosely constrained parameter. Real-time GPS clocks provided by the NASA Global Differential GPS System [www.gdgps.net] is of 10–20 cm accuracy. Although their effects are less significant, these clocks are also estimated as loosely constrained parameters.

Tables 1 summarizes all the estimated parameters associating with the orbital states, with their pre-specified constraints. The initial constraint on each process-noise empirical force consists of an initial and a steady-state constraints, separated by a slash (/) in the table. Table 2 includes all other estimated parameters and the constraints to be applied. Fig. 1 describes the flow of the entire estimation process. New epochs in the flow are defined by GPS data.

Table 1. Orbit State Parameters to be Estimated

	Reference Orbit	Differential Orbit
Orbit Position (m)	1	1
Orbit Velocity (mm/s)	1	1
Solar Radiation Scaling	1	0.1
Drag Coefficient	1	0.1
Empirical Force, modeled as process noise:		
Radial (nm/sec^2)	30 / 30	0.5 / 0.1
Cross-Track (nm/sec^2)	30 / 30	1.0 / 0.3
In-Track (nm/sec^2)	30 / 30	3.0 / 1.0
Correlation Time (s)	5400	600

After the UD array is updated by the successfully fixed biases at each epoch, the differential and reference orbit state vectors are updated with the adjustment estimate (embedded in UD array) before being mapped to the next epoch. Such state-vector updates characterize an extended Kalman filter and are essential for proper convergence of orbit solutions in real-time applications. Lacking such state-vector updates, orbit solutions will converge only with precise initialization and dynamics modeling.

Table 2. Other Parameters to be Estimated

Differential User Clock (randomwalk)	1 m + 1 cm/s$^{1/2}$
GPS Clocks (randomwalk)	1 m + 1 cm/(10s)$^{1/2}$
Ionosphere Delays (white-noise)	100 m @ 1Ghz
Phase Biases (constant over each continuous link)	10 km

With a straightforward implementation, the total number of estimated parameters can be large. For instance, with 30 GPS satellites there will be 30 GPS clocks, 60 ionosphere delays, 120 phase biases, in addition to 1 for the user clock offset and 11 for each of the state vectors of the differential and the reference orbits, with a total of 233. The number can be greatly reduced by removing those GPS satellites that are not actually involved at any epoch. However, such an epoch-dependent filter size will complicate the bookkeeping in modeling the estimated parameters. A compromise is adopted instead. Since not more than half of the GPS satellites are in view at any epoch, we can assume only 15 potentially in-view GPS satellites at all epochs. In this way, the total number of estimated parameters will be reduced by nearly a factor of 2 and the processing time for the filtering process will be shortened by a factor of nearly 4. At each epoch, a drop-out GPS satellite will be flagged as inactive; but its associating parameters will remain in the estimating list until being replaced by those associated with a newly acquired GPS satellite.

DEMONSTRATION WITH GRACE

To demonstrate the real-time differential orbit determination described above, L1 and L2 pseudorange and carrier phase data from the two GRACE onboard GPS receivers are used. GRACE is a **G**ravity **R**ecovery **A**nd **C**limate **E**xperiment mission involving two satellites flying in tandem at a nominal height of 500 km and separated by ~200 km. In addition to the onboard GPS receivers for cm-level orbit determination, a dual-one-way **K**-**B**and **R**anging system (KBR) provides micron-level inter-satellite distance determination. The KBR solution is ideal for certifying the differential orbit accuracy with GPS bias fixing.

The main data set covers the first 9 days of August 2005 at a data sampling interval of 10 sec. In December 2005 the two spacecraft switch orbital location, and during this maneuver the inter-spacecraft range was reduced to nearly zero. We have also processed data from this period of time, with baseline length varying from 65 km to 35 km.

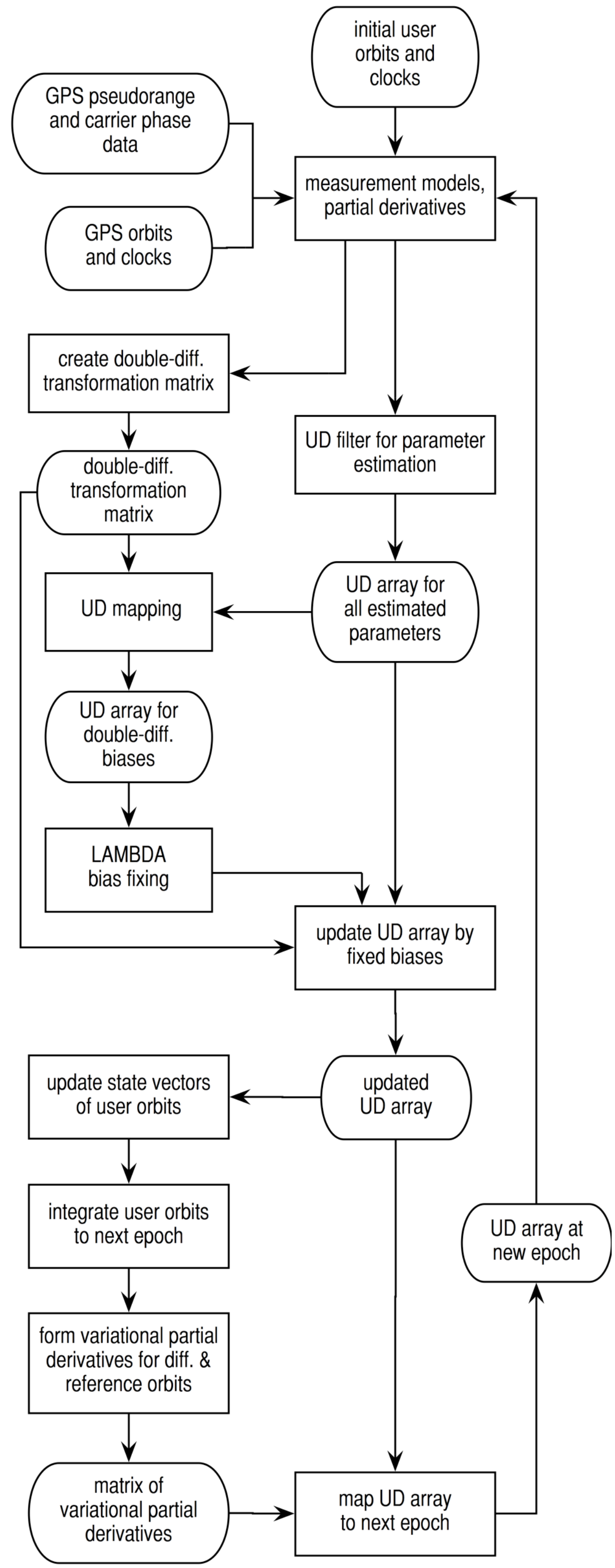

Fig. 1. Flow diagram of estimation process

The GPS data are weighted at 15 cm for pseudorange and 2 mm for the carrier phase. Also used are the GRACE attitude information from the onboard star cameras. The real-time solutions of the GPS orbit and clock states were provided by the JPL GDGPS system, using the Real Time GIPSY (RTG) software. They are accurate to 10-20 cm [www.gdgps.net]. Only crude (accurate to a few meters) GRACE orbits are needed to initiate the orbit integrations. Dynamic models for the integrations include a 200x200 gravity, atmospheric drag, solar radiation and tides, and 3-d empirical forces, as described in Table 1.

RESULTS OF DEMONSTRATION

Two aspects of the quality of the demonstration results are examined. First, the number of L1/L2 pairs of successfully fixed double-differenced biases over the entire data span provides a qualitative measure of the filtering process. Secondly, the ultimate accuracy of the resulting differential orbit is assessed by comparing to the precise (to the micron level) K-band dual one-way ranging solution. A sample 1-day results are shown in Figs. 2 and 3. In Fig. 2, the upper frame shows the actual number of L1/L2 bias pairs successfully fixed while the lower frame the success rate. At the beginning of the day, some components of the formal error of the differential position estimate are higher than the 2-cm threshold, thus no bias fixing is intended. Such initializing period lasts for ~20 minutes. Beyond this initializing period, about 80% of the double-differenced biases are successfully fixed. The results for other days are similar.

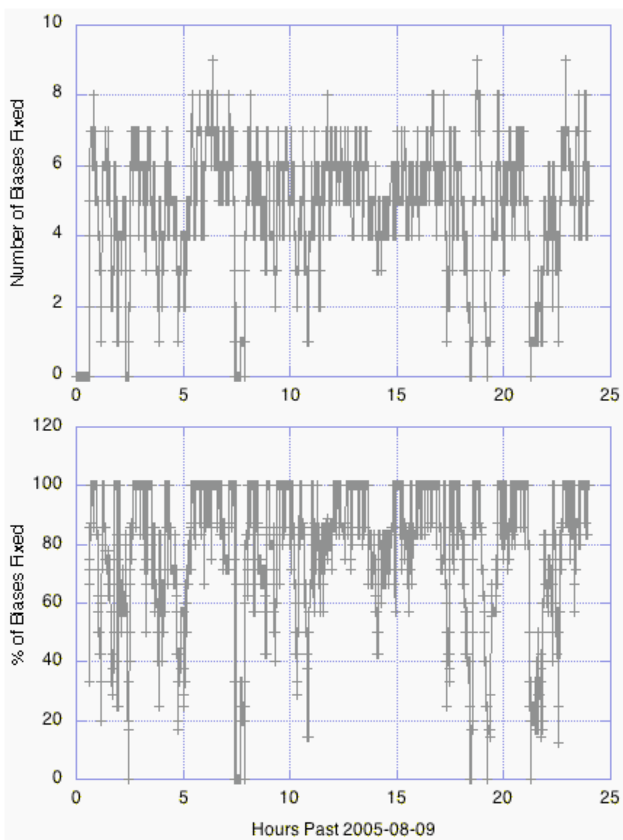

Fig. 2. Number of L1/L2 pairs of double-differenced biases fixed to integer values

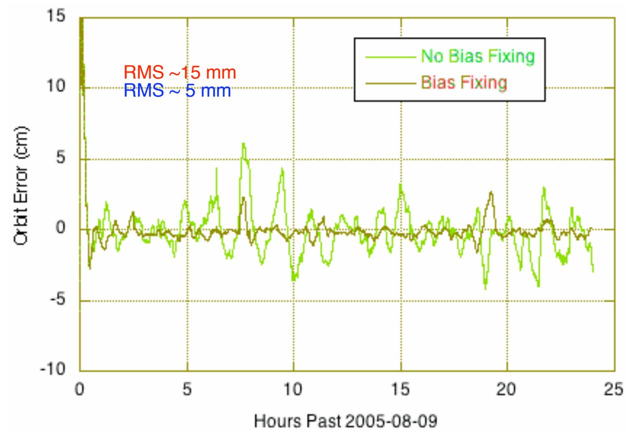

Fig. 3. Differential orbit accuracy (deviation from K-band dual one-way ranging solution)

In Fig. 3, the quality of the 1-day solution for the differential orbit is shown. Here, the deviations of the inter-satellite distance solutions, with and without bias fixing, from the KBR solution are compared.

At the beginning of the day when no biases can be fixed, the differential orbit solution is worse. After about 20 minutes when biases are getting fixed, the orbit solutions greatly improves and has an RMS value of ~5 mm for the remaining of the day. The superiority of the solution over that without bias fixing is clearly observed. Although only the inter-satellite distance accuracy can be assessed, the other two components are likely to be of similar accuracy.

Fig. 4 compares the daily RMS differential orbit accuracy with different weighting on GPS phase data: a nominal weight (2 mm), a 1.5 times heavier weight (1.33 mm) and a 2 times heavier weight (1 mm), for the entire 9-day

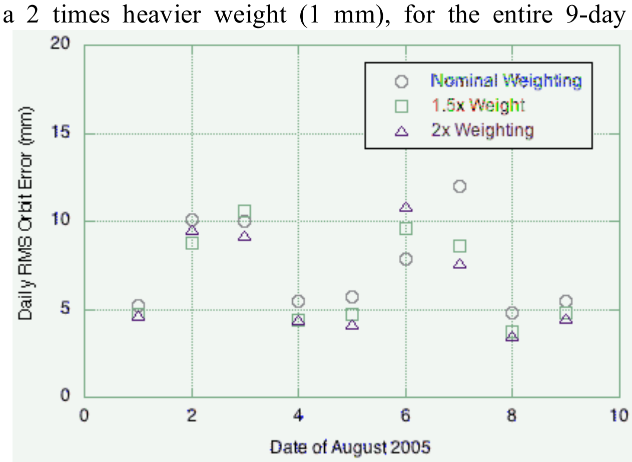

Fig. 4. Differential orbit accuracy (deviation from K-band dual one-way ranging solution) with different weighting on GPS phase data

period. Pseudorange data is weighted at 15 cm throughout. It is observed that the accuracy is not quite sensitive to the relative weighting, with the heavier phase data weights yielding slightly better solution. The RMS orbit accuracy with these heavier weights is ~ 7 mm (1D RMS). The ~20-minute initializing period before bias fixing gets started at the beginning of each day has been excluded.

Fig. 5 shows the corresponding percentage of biases successfully fixed for the cases studied as in Fig. 4. The same low sensitivity to relative data weighting is observed. The overall average success rate of bias fixing is 79% over the 9-day period.

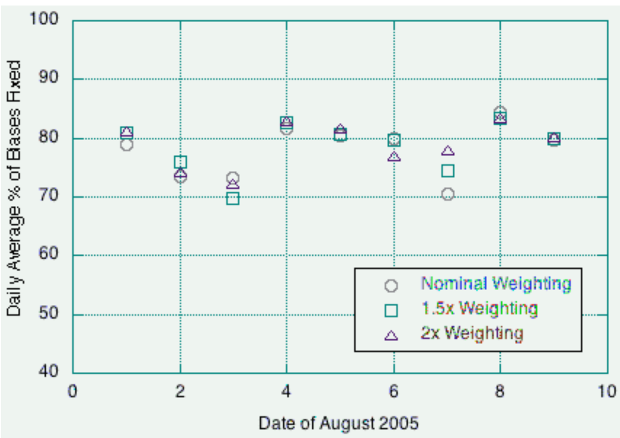

Fig. 5. Daily average percentage of successfully fixed biases with different weighting on GPS phase data

Over the shorter baselines of December 2005, varying from 65 to 35 km, the relative positioning accuracy slightly improved to ~4 mm 1 D RMS. A comparison of results with different baseline length is as shown in Fig. 6. The better accuracy is due partly to better cancellation of dynamics and GPS orbit error between the two ends of the shorter baseline.

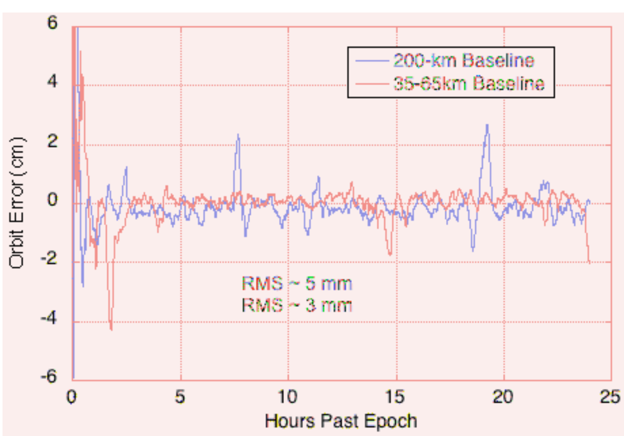

Fig. 6. Better orbit accuracy with shorter baselines

CONCLUDING REMARKS

An effective UD formulated estimation technique for precise real-time differential orbit determination of formation flying low earth satellites has been proposed. The high accuracy results from integer-valued fixing of GPS carrier phase biases with high success rate. Even though it is double-differenced biases that are to be fixed, no explicit differencing of GPS data is needed. A transformation matrix converts the UD array for un-differenced biases into that for double-differenced biases. The same transformation matrix also serves as the left-hand side of measurement equations for updating the UD array for integer-value fixed double-differenced biases.

State vectors of the differential LEO orbit and a reference orbit, instead of individual LEO orbits, are estimated to exploit the high commonality of orbit dynamics. In this way the constraints on the estimated dynamic parameters can be tightly set so as to strengthen the orbit estimation. Measurement partial derivatives with respect to state vectors of differential orbit and of reference orbit can be easily derived from those of individual orbits with the chain rule. So can the matrix of variational partial derivatives mapping the state vectors and the associating UD array from one epoch to the next.

A demonstration has been performed for the two GRACE satellites, separated by a distance of ~200 km, over a 9-day period. The results of this demonstration has shown that ~80% of double-differenced biases are successfully fixed. The differential orbit accuracy were assessed by comparing with the precise (to the micron level) K-band dual one-way ranging. The component in the line-of-sight direction has a ~7 mm agreement. Though not readily comparable, the other components are believed to have similar accuracy. Over shorter baselines (35-65 km) the accuracy improved to ~4 mm.

The quality of the real-time GPS orbit and clock states becomes less important as the inter-spacecraft baseline gets shorter. Over the nominal 200 km GRACE baseline it proved difficult to resolve ambiguities with the broadcast ephemeris (~1 m URE), but even without bias-fixing the relative positioning of the two spacecraft was at the few centimeter level. Over shorter baselines it became possible to fix the integer ambiguity with the GPS broadcast ephemeris, and to obtain sub-cm relative positioning.

Although the estimation technique has been demonstrated with an application containing only two satellites, it is readily applicable to multi-satellite differential orbit determination. The corresponding transformation matrix forming double-differenced biases, though more complicated, has been implemented.

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